

Family Gauge Symmetry and Koide's Mass Formula

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Koide's mass formula is an empirical relation among the charged lepton masses which holds with a striking precision. We propose a mechanism for cancelling the QED correction to Koide's formula. This is discussed in an effective theory with $U(3)$ family gauge symmetry and a scenario in which this symmetry is unified with $SU(2)_L$ symmetry at 10^2 – 10^3 TeV scale.

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Among various properties of elementary particles, the spectra of the quarks and leptons exhibit unique patterns, and their origin still remains as a profound mystery. Koide's mass formula is an empirical relation among the charged lepton masses given by [1]

$$\frac{\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}}{\sqrt{m_e + m_\mu + m_\tau}} = \sqrt{\frac{3}{2}}, \quad (1)$$

which holds with a striking precision. In fact, substituting the present experimental values of the charged lepton masses [2], the formula is valid within the present experimental accuracies. The relative experimental error of the left-hand side of eq. (1) is of order 10^{-5} .

Given the remarkable accuracy with which eq. (1) holds, there have been many speculations as to existence of some physical origin behind this mass formula [1, 3]. Despite the attempts to find its origin, so far no realistic model or mechanism has been found which predicts Koide's mass formula within the required accuracy. The most serious problem one faces in finding a realistic model or mechanism is caused by the QED radiative correction. Even if one postulates some mechanism at a high energy scale that leads to this mass relation, the charged lepton masses receive the 1-loop QED radiative correction given by

$$m_i^{\text{pole}} = \left[1 + \frac{\alpha}{\pi} \left\{ \frac{3}{4} \log \left(\frac{\mu^2}{\bar{m}_i(\mu)^2} \right) + 1 \right\} \right] \bar{m}_i(\mu), \quad (2)$$

where $\bar{m}(\mu)$ and m^{pole} denote the running mass defined in the modified-minimal-subtraction scheme ($\overline{\text{MS}}$ scheme) and the pole mass, respectively; μ represents the renormalization scale. It is the pole mass that is measured in experiments. Suppose $\bar{m}_i(\mu)$ (or the corresponding Yukawa couplings $\bar{y}_i(\mu)$) satisfy the relation (1) at scale $\mu \gg M_W$. Then m_i^{pole} do not satisfy the same relation [5]: Eq. (1) is corrected by approximately 0.1%, which is 120 times larger than the present experimental error. Note that this correction originates only from the term $-3\alpha/(4\pi) \times \bar{m}_i \log(\bar{m}_i^2)$ of eq. (2), since the other terms, which are of the form $\text{const.} \times \bar{m}_i$, do not affect the relation (1). We also note that $\log(\bar{m}_i^2)$ results from the fact that \bar{m}_i plays a role of an infrared cut-off in the loop integral.

The 1-loop weak correction is of the form $\text{const.} \times \bar{m}_i$ in the leading order of \bar{m}_i^2/M_W^2 expansion; the leading

non-trivial correction is $\mathcal{O}(G_F \bar{m}_i^3/\pi)$ whose effect can be safely neglected. Other known radiative corrections are also negligible.

Thus, if there is indeed a physical origin to Koide's mass formula at a high energy scale, we need to account for a correction to the relation (1) that cancels the QED correction. Since such a correction is absent up to the scale of $\mathcal{O}(M_W)$ to our present knowledge, it must originate from a higher scale. Then, there is a difficulty in explaining why the size of such a correction should coincide accurately with the size of the QED correction which arises from much lower scales. Up to date, no mechanism has been proposed to solve this problem.

Among various existing models which attempt to explain origins of Koide's mass formula, we find a class of models particularly attractive [6]. These are the models which predict the mass matrix of the charged leptons to be proportional to the square of the vacuum expectation value (VEV) of a scalar field (we denote it as Φ) written in a 3-by-3 matrix form:

$$\mathcal{M}_\ell \propto \langle \Phi \rangle \langle \Phi \rangle. \quad (3)$$

Thus, $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ is proportional to the diagonal elements of $\langle \Phi \rangle$ in the basis where it is diagonal. The VEV $\langle \Phi \rangle$ is determined by minimizing the potential of scalar fields in each model. Hence, the origin of Koide's formula is attributed to the specific form of the potential which realizes this relation in the vacuum configuration. Up to now, no model is complete with respect to symmetry: Every model requires either absence or strong suppression of some of the terms in the potential (which are allowed by the symmetry of that model), without justification.

In this paper, we propose a possible mechanism for cancelling the QED correction to Koide's mass formula, in the context of a theory with family (horizontal) gauge symmetry. In our study we adopt the mechanism eq. (3) for generating the charged lepton masses at tree level, due to the following reasons. First, models with this mechanism are suited for perturbative analyses. Secondly, since Φ is renormalized multiplicatively, the structure of radiative corrections is simple.

We consider an effective theory with family gauge symmetry which is valid up to some cut-off scale denoted by Λ ($\gg M_W$). Within such an effective theory, the charged

lepton masses may be induced by a higher-dimensional operator

$$\mathcal{O} = \frac{\kappa(\mu)}{\Lambda^2} \bar{\psi}_{Li} \Phi_{ik} \Phi_{kj} \varphi e_{Rj}. \quad (4)$$

Here, $\psi_{Li} = (\nu_{Li}, e_{Li})^T$ denotes the left-handed lepton $SU(2)_L$ doublet of the i -th generation; e_{Rj} denotes the right-handed charged lepton of the j -th generation; φ denotes the Higgs doublet field; Φ is a 9-component scalar field and is singlet under the Standard Model (SM) gauge group. We suppressed all the indices except for the generation (family) indices $i, j, k = 1, 2, 3$. (Summation over repeated indices is understood throughout the paper.) The dimensionless Wilson coefficient of this operator is denoted as $\kappa(\mu)$. Once Φ acquires a VEV, the operator \mathcal{O} will effectively be rendered to the Yukawa interactions of the SM; after the Higgs field also acquires a VEV, $\langle \varphi \rangle = (0, v_{ew}/\sqrt{2})^T$ with $v_{ew} \approx 250$ GeV, the operator will induce the charged-lepton mass matrix of the form eq. (3) at tree level:

$$\mathcal{M}_\ell^{\text{tree}} = \frac{\kappa v_{ew}}{\sqrt{2}\Lambda^2} \langle \Phi \rangle \langle \Phi \rangle. \quad (5)$$

We consider radiative corrections to the above mass matrix by the family gauge interaction. First we consider the case, in which the gauge group is $SU(3)$ and both ψ_L and e_R are assigned to the $\mathbf{3}$ (fundamental representation) of this symmetry group. With this choice of representation, however, Koide's formula is subject to a severe radiative correction unless the family gauge interaction is strongly suppressed. In fact, the 1-loop correction by the family gauge bosons induces an effective operator

$$\mathcal{O}' \sim \frac{\alpha_F}{\pi} \times \kappa \bar{\psi}_{Li} \varphi e_{Ri} \times \frac{\langle \Phi \rangle_{jk} \langle \Phi \rangle_{kj}}{\Lambda^2}, \quad (6)$$

hence corrections universal to all the charged-lepton masses, $(\delta m_e, \delta m_\mu, \delta m_\tau) \propto (1, 1, 1)$, are induced. This is due to the fact that the dimension-4 operator $\bar{\psi}_{Li} \varphi e_{Ri}$ is not prohibited by symmetry. Here, $\alpha_F = g_F^2/(4\pi)$ denotes the gauge coupling constant of the family gauge interaction. As noted above, corrections which are proportional to individual masses do not affect Koide's formula; oppositely, the universal correction violates Koide's formula strongly. In order that the correction to Koide's formula cancel the QED correction, a naive estimate shows that α_F/π should be order 10^{-5} , provided that the cut-off Λ is not too large and that the above operator \mathcal{O}' is absent at tree level. If \mathcal{O}' exists at tree level, there should be a fine tuning between the tree and 1-loop contributions. The situation is similar if the family symmetry is $O(3)$ and both ψ_L and e_R are in the $\mathbf{3}$. In these cases [7] we were unable to find any sensible reasoning for the cancellation between the QED correction and the correction induced by family gauge interaction, other than to regard the cancellation as just a pure coincidence. Hence, we will not investigate these choices of representation further.

In the case that ψ_L is assigned to $\mathbf{3}$ and e_R to $\bar{\mathbf{3}}$ (or *vice versa*) of $U(3)$ family gauge group, (i) the dimension-4 operator $\bar{\psi}_{Li} \varphi e_{Ri}$ is prohibited by symmetry, and hence corrections universal to all the three masses do not appear; and (ii) marked resemblance of the radiative correction to the QED correction follows. We show these points explicitly in a specific setup.

We denote the generators for the fundamental representation of $U(3)$ by T^α ($0 \leq \alpha \leq 8$), which satisfy

$$\text{tr}(T^\alpha T^\beta) = \frac{1}{2} \delta^{\alpha\beta} \quad ; \quad T^\alpha = T^{\alpha\dagger}. \quad (7)$$

$T^0 = \frac{1}{\sqrt{6}} \mathbf{1}$ is the generator of $U(1)$, while T^a ($1 \leq a \leq 8$) are the generators of $SU(3)$.

We assign ψ_L to the representation $(\mathbf{3}, 1)$, where $\mathbf{3}$ stands for the $SU(3)$ representation and 1 for the $U(1)$ charge, while e_R is assigned to $(\bar{\mathbf{3}}, -1)$. Under $U(3)$, the 9-component field Φ transforms as three $(\mathbf{3}, 1)$'s. Explicitly the transformations of these fields are given by

$$\psi_L \rightarrow U \psi_L, \quad e_R \rightarrow U^* e_R, \quad \Phi \rightarrow U \Phi \quad (8)$$

with $U = \exp(i\theta^\alpha T^\alpha)$.

We assume that the charged-lepton mass matrix is induced by a higher-dimensional operator $\mathcal{O}^{(\ell)}$ similar to \mathcal{O} in eq. (4). We further assume that $\langle \Phi \rangle$ can be brought to a diagonal form in an appropriate basis. Thus, in this basis $\mathcal{O}^{(\ell)}$, after Φ and φ acquire VEVs, turns to the lepton mass terms as

$$\mathcal{O}^{(\ell)} \rightarrow \frac{\kappa^{(\ell)}(\mu) v_{ew}}{\sqrt{2}\Lambda^2} \bar{\psi}_L \Phi_d(\mu)^2 e_R, \quad (9)$$

where

$$\Phi_d(\mu) = \begin{pmatrix} v_1(\mu) & 0 & 0 \\ 0 & v_2(\mu) & 0 \\ 0 & 0 & v_3(\mu) \end{pmatrix}, \quad v_i(\mu) > 0. \quad (10)$$

When all v_i are different, $U(3)$ symmetry is completely broken by $\langle \Phi \rangle = \Phi_d$, and the spectrum of the $U(3)$ gauge bosons is determined by Φ_d .

Note that the operator \mathcal{O} in eq. (4) is *not* invariant under the $U(3)$ transformations eq. (8). As an example of $\mathcal{O}^{(\ell)}$, one may consider

$$\mathcal{O}_1^{(\ell)} = \frac{\kappa^{(\ell)}(\mu)}{\Lambda^2} \bar{\psi}_L \Phi \Phi^T \varphi e_R. \quad (11)$$

It is invariant under a larger symmetry $U(3) \times O(3)$, under which Φ transforms as $\Phi \rightarrow U \Phi O^T$ ($O O^T = \mathbf{1}$). In this case, we need to assume, for instance, that the $O(3)$ symmetry is gauged and spontaneously broken at a high energy scale before the breakdown of the $U(3)$ symmetry, in order to eliminate Nambu-Goldstone bosons and to suppress mixing of the $U(3)$ and $O(3)$ gauge bosons. In any case, the properties of $\mathcal{O}^{(\ell)}$ given by eqs. (9) and (10) are sufficient for computing the radiative correction by the $U(3)$ gauge bosons to the mass matrix, without an explicit form of $\mathcal{O}^{(\ell)}$.

We take the $U(1)$ and $SU(3)$ gauge coupling constants to be the same:

$$\alpha_{U(1)} = \alpha_{SU(3)} = \alpha_F. \quad (12)$$

We compute the radiative correction in Landau gauge, which is known to be convenient for computations in theories with spontaneous symmetry breaking. Through standard computation, one obtains

$$\delta m_i^{\text{pole}} = -\frac{3\alpha_F}{8\pi} \left[\log \left(\frac{\mu^2}{v_i(\mu)^2} \right) + c \right] m_i(\mu), \quad (13)$$

$$m_i(\mu) = \frac{\kappa^{(\ell)}(\mu) v_{\text{ew}}}{\sqrt{2}\Lambda^2} v_i(\mu)^2. \quad (14)$$

Here, c is a constant independent of i . The Wilson coefficient $\kappa^{(\ell)}(\mu)$ is defined in $\overline{\text{MS}}$ scheme. $v_i(\mu)$ are defined as follows: The VEV of Φ at renormalization scale μ , $\Phi_d(\mu) = \langle \Phi(\mu) \rangle$ given by eq. (10), is determined by minimizing the 1-loop effective potential in Landau gauge (although we do not discuss the explicit form of the effective potential); Φ is renormalized in $\overline{\text{MS}}$ scheme. We ignored terms suppressed by $m_i^2/v_j^2 (\ll 1)$ in the above expression. Note that the pole mass is renormalization-group invariant and gauge independent. Therefore, the above expression is rendered gauge independent if we express $v_i(\mu)$ in terms of gauge independent parameters, such as coupling constants defined in on-shell scheme.

The form of the radiative correction given by eqs. (13) and (14) is constrained by symmetries and their breaking patterns. As the diagonal elements of the VEV, $v_3 > v_2 > v_1 > 0$, are successively turned on, gauge symmetry is broken according to the pattern:

$$U(3) \rightarrow U(2) \rightarrow U(1) \rightarrow \text{nothing}. \quad (15)$$

At each stage, the gauge bosons corresponding to the broken generators acquire masses and decouple. Furthermore, the vacuum Φ_d and the family gauge interaction respect a global $U(1)^3$ symmetry generated by

$$\psi_L \rightarrow V \psi_L, \quad e_R \rightarrow V^* e_R, \quad \Phi_d \rightarrow V \Phi_d V^*, \quad (16)$$

with

$$V = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix} \quad ; \quad \phi_i \in \mathbf{R}. \quad (17)$$

Although $\mathcal{O}^{(\ell)}$ after symmetry breakdown, eq. (9), is not invariant under this transformation, the variation can be absorbed into a redefinition of v_i . As a result, the lepton mass matrix has a following transformation property:

$$\mathcal{M}_\ell \Big|_{v_i \rightarrow v_i \exp(i\phi_i)} = V \mathcal{M}_\ell V^*. \quad (18)$$

This is satisfied including the 1-loop radiative correction. The symmetry breaking pattern eq. (15) and the above

transformation property constrain the form of the radiative correction to $\delta m_i^{\text{pole}} \propto v_i^2 [\log(|v_i|^2) + \text{const.}]$, where the constant is independent of i . Note that $|v_i|^2$ in the argument of logarithm originates from the gauge boson masses, which are invariant under $v_i \rightarrow v_i \exp(i\phi_i)$.

The universality of the $SU(3)$ and $U(1)$ gauge couplings eq. (12) is necessary to guarantee the above symmetry breaking pattern eq. (15). One may worry about validity of the assumption for the universality, since the two couplings are renormalized differently in general. The universality can be ensured approximately if these two symmetry groups are embedded into a simple group down to a scale close to the relevant scale. There are more than one ways to achieve this. A simplest way would be to embed $SU(3) \times U(1)$ into $SU(4)$. It is easy to verify that the **4** of $SU(4)$ decomposes into $(\mathbf{3}, -\frac{1}{2}) \oplus (\mathbf{1}, \frac{3}{2})$ under $SU(3) \times U(1)$. Hence, the **6** (second-rank antisymmetric representation) and $\bar{\mathbf{6}}$ of $SU(4)$, respectively, include $(\bar{\mathbf{3}}, -1)$ and $(\mathbf{3}, 1)$.

Within the effective theory under consideration, the QED correction to the pole mass is given just as in eq. (2) with $\bar{m}_i(\mu)$ replaced by $m_i(\mu)$. Recall that corrections of the form $\text{const.} \times m_i$ do not affect Koide's formula. Noting $\log v_i^2 = \frac{1}{2} \log m_i^2 + \text{const.}$, one observes that if a relation between the QED and family gauge coupling constants

$$\alpha = \frac{1}{4} \alpha_F \quad (19)$$

is satisfied, the 1-loop radiative correction induced by family gauge interaction cancels the 1-loop QED correction to Koide's mass formula.

Suppose the relation (19) is satisfied. Then

$$m_i^{\text{pole}} \propto v_i(\mu)^2 \quad (20)$$

holds with a good accuracy. This is valid for any value of μ . This means, if $v_i(\mu)$ satisfy

$$\frac{v_1(\mu) + v_2(\mu) + v_3(\mu)}{\sqrt{v_1(\mu)^2 + v_2(\mu)^2 + v_3(\mu)^2}} = \sqrt{\frac{3}{2}} \quad (21)$$

at some scale μ , Koide's formula is satisfied at any scale μ . This is a consequence of the fact that Φ is multiplicatively renormalized. Generally, the form of the effective potential varies with scale μ . If the relation (21) is realized at some scale as a consequence of a specific nature of the effective potential (in Landau gauge), the same relation holds automatically at any scale. Although these statements are formally true, physically one should consider scales only above the family gauge boson masses, since decoupling of the gauge bosons is not encoded in $\overline{\text{MS}}$ scheme. An interesting possibility is to use eq. (20) to relate the charged lepton pole masses with the VEV at the cut-off scale, i.e. $\mu = \Lambda$, which sets a boundary (initial) condition of the effective theory.

The advantages of choosing Landau gauge in our computation are two folds: (1) The computation of the 1-loop effective potential becomes particularly simple (as

well known in computations of the effective potential in various models); in particular there is no $\mathcal{O}(\alpha_F)$ correction to the effective potential. (2) The lepton wave-function renormalization is finite; as a consequence, the $p_\mu \gamma^\mu$ part of the lepton self-energy is independent of generation. Due to the former property, there is no $\mathcal{O}(\alpha_F)$ correction to the relation eq. (21) if it is satisfied at tree level. Due to the latter property, the correction to Koide's formula is determined by renormalization of $\mathcal{O}^{(\ell)}$ alone, and a simple relation to $\langle \Phi(\mu) \rangle$ follows.

Let us comment on gauge dependence of our prediction. If we take another gauge and express the radiative correction δm_i^{pole} in terms of $\langle \Phi(\mu) \rangle$, the coefficient of $\log(\mu^2/\langle \Phi \rangle^2)$ changes, and other non-trivial flavor dependent corrections are induced. Suppose the relation eq. (21) is satisfied at tree level [11]. The VEV $\langle \Phi \rangle$ in another gauge receives an $\mathcal{O}(\alpha_F)$ correction, which induces a correction to eq. (21) at $\mathcal{O}(\alpha_F)$. These additional corrections to δm_i^{pole} at $\mathcal{O}(\alpha_F)$ should cancel altogether if they are reexpressed in terms of the tree-level v_i 's which satisfy eq. (21), since the $\mathcal{O}(\alpha_F)$ correction to the relation (21) vanishes in Landau gauge. General analyses on gauge dependence of the effective potential may be found in [9].

Now we speculate on a possible scenario how the relation (19) may be satisfied. The scale of α is determined by the charged lepton masses, while the scale of α_F is determined by the family gauge boson masses, which should be much higher than the electroweak scale. Since the relevant scales of the two couplings are very different, we are unable to avoid assuming some accidental factor (or parameter tuning) to achieve this condition. Instead we seek for an indirect evidence which indicates such an accident has occurred in Nature. The relation (19) shows that the value of α_F is close to that of the weak gauge coupling constant α_W , since $\sin^2 \theta_W(M_W)$ is close to $1/4$. In fact, within the SM, $\frac{1}{4} \alpha_W(\mu)$ approximates $\alpha(m_\tau)$ at scale $\mu \sim 10^2\text{--}10^3$ TeV. Hence, if the electroweak $SU(2)_L$ gauge group and the $U(3)$ family gauge group are uni-

fied around this scale, naively we expect that $\alpha \approx \frac{1}{4} \alpha_F$ is satisfied. Since α_W runs relatively slowly in the SM, even if the unification scale is varied within a factor of 3, Koide's mass formula is satisfied within the present experimental accuracy. This shows the level of parameter tuning required in this scenario.

We assume that anomalies introduced by the couplings of fermions to family gauge bosons are cancelled, which requires existence of fermions other than the SM fermions. Furthermore, we assume that all the additional fermions acquire masses of order $\langle \Phi \rangle$ or larger after the spontaneous breakdown of $U(3)$, so that they decouple from the SM sector at and below the electroweak scale.

A characteristic prediction of the present scenario is the existence of lepton flavor violating processes at $10^2\text{--}10^3$ TeV scale. For instance, assuming that the down-type quarks are in the same representation of $U(3)$ as the charged leptons, and that the mass matrices of the charged leptons and down-type quarks are simultaneously diagonalized in an appropriate basis, we find $\Gamma(K_L \rightarrow e\mu) \approx m_\mu^2 m_{K_L} f_K^2 / (16\pi v_2^4)$. Comparing to the present experimental bound $\text{Br}(K_L \rightarrow e\mu) < 4.7 \times 10^{-12}$ [2], we obtain a limit $v_2 \gtrsim 5 \times 10^2$ TeV. Naively this limit may be marginally in conflict with the estimated unification scale in the above scenario. This depends, however, rather heavily on our assumptions on the quark sector. In the case that there exist additional factors in the quark sector which suppress the decay width by a few orders of magnitude, we may expect a signal for $K_L \rightarrow e\mu$ not far beyond the present experimental reach. Predictions concerning purely leptonic processes are less model dependent, but expected event rates are far below present experimental sensitivities.

Models, which predict a realistic charged lepton spectrum incorporating the mechanism proposed in this paper, will be discussed elsewhere [10].

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